Section 4: Andrew Wiles

With the Taniyama-Shimura Conjecture, the connection between the elegant elliptic curves and the complex modular forms was established. By the mid-1980's it was still unproven. It was interesting and hard to believe, but not of immediate concern. But what did it have to do with Fermat's Last Theorem? Seemingly ridiculous statements were becoming more common and in 1984 German mathematician Gerhard Frey made another one: if Taniyama-Shimura conjecture was true, so was Fermat's last Theorem. What did Fermat's Last Theorem have to do with elliptic curves/modular forms?

Frey had done something remarkable. He asked an unthinkable question: what if Fermat was wrong? After 350 years or so, mathematicians had essentially determined that his theorem was correct but he did not really have a correct proof. But what if he was wrong and $x^n + y^n = z^n$ had a solution for some $n \geq 3$? Suppose there actually WAS a solution (A, B, C) to some such equation. Frey used these numbers to create an elliptic curve, now called Frey curves:

$$E_{A,B} : y^2 = x(x + A^n)(x - B^n).$$

So if there was a solution to Fermat's Last Theorem, we had this new elliptic curve. In studying these hypothetical elliptic curves, Frey noticed they were very strange. In fact, they appeared to have one VERY strange property: they were not modular. Recall that the conjecture of Taniyama-Shimura stated that all elliptic curves were modular. Frey's conjecture said that Frey's elliptic curves were not modular. So if Taniyama-Shimura was correct, Frey's curves could not exist. Hence FLT could not have any solutions.

The conjecture eventually became known as the Epsilon Conjecture (within a couple of years after Frey stated his conjecture, Jean-Pierre Serre proved he was correct for "all but ε"). Frey had formulated his theory, curves, and conjecture after studying with Barry Mazur at Harvard in the early 1980's. Once he announced his idea, Ken Ribet, another colleague of Mazur, thought at first it was a joke. But when he saw Serre's work, he soon recognized in it something he had already considered himself. He was able to modify his previous research and by 1990 he proved the Epsilon Conjecture. Frey curves were definitely not modular.

So all that remained to do was prove Taniyama-Shimura. Enter Andrew Wiles. Wiles' story is an interesting, compelling, and heartwarming one. As a child, he had long been fascinated by Fermat's Last Theorem. The simplicity of the equation $x^n + y^n = z^n$, the relationship to the Pythagorean theorem, the amazing depth of the conclusion had intrigued him. Once he entered graduate school, it was already considered most likely true, and yet most respected mathematicians thought it foolish to try and prove it. At that time (the 1960's) no one had any idea where to begin and every great mathematician since the 17th century had tried and failed. So he abandoned his first mathematical love and went into the mathematical field of his major professor John Coates: elliptic curves.

In 1990, Wiles was relaxing at a friend's house when suddenly his friend said "By the way, did you hear that Ken Ribet proved the Epsilon Conjecture?" Wiles relates that he was "electrified" and he knew that his life was changing its course at that moment. Ribet had made it possible to prove FLT by working in elliptic curves, the field serendipity had led Wiles into instead. Wiles set out to prove Taniyama-Shimura in an effort to solve the problem he loved since childhood.

It took several years, a lot of solitary work, and a few collaborations with colleagues, but in 1994, Wiles accomplished his life's goal. He announced his proof of the Taniyama-Shimura Conjecture in the summer of 1993 and by the fall of 1994, after fixing a few errors, the final proof was accepted. Fermat's Last Theorem will in some ways always be known by that name, but gradually another name is becoming more common:

Theorem 3.4.1 (Wiles' Theorem) The equation $x^n + y^n = z^n$ has no nontrivial solutions if $n \geq 3$.